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MODEL-BASED ACOUSTIC ARRAY PROCESSING

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Abstract: *Model-Based Processing is essentially a way of incorporating physics into the processing scheme in a self-consistent manner. This work presents some of the techniques that have been applied to acoustic array problems. Three situations are addressed. The first is the bearing estimation problem. It is shown that if the forward motion of a towed array is incorporated into the signal model, the performance, as measured by the variance of the estimate, is significantly improved. The second problem treated is that of range estimation. Here it is shown that, by modeling the signal as a cylindrical wavefront, and including the forward motion of the array, the range of an acoustic source can be estimated with an array whose physical aperture is short as compared to the range of the source. The third problem addressed is that of model-based localization of a source using a fixed vertical array. In this case, the signal is represented by a normal-mode propagation model. This differs from matched field processing in that it includes the propagation model parameters themselves in the scheme, thereby dealing with the so-called "mismatch" problem, i.e., the problem that arises when the model parameters are not well-known. It also differs from the matched-field approach in that it does not require an exhaustive search over the parameters of interest to obtain a solution. The performance improvements that Model-Based Processing is capable of are demonstrated using experimental results.*

Keywords: *Arrays, Model-Based, Synthetic Aperture, Localization*

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1. INTRODUCTION

Model-Based processing refers to the practice of including the physics of the situation in the processing scheme for the purpose of improving performance. An early example is the case of source localization in the ocean, commonly referred to as “Matched-Field Processing” or MFP [1, 2]. It is a localization scheme that incorporates a propagation model into the processor, thereby improving the observability [3] of the parameters of interest. In this paper it is shown, for three cases, how this philosophy can improve performance over conventional approaches.

The first case is that of bearing estimation with a line array. By explicitly including the forward motion of the array in the processor, the variance on the bearing estimate is significantly reduced. This is referred to as the “Synthetic Aperture Effect,” since the improvement can be heuristically viewed as exploiting the aperture traced out by the moving array. In fact, the improvement arises from the fact that the model-based approach allows the bearing information contained in the Doppler shift of the received data to be exploited.

The second case is the estimation of range by “Wavefront-Curvature Ranging.” In this case, along with the array motion, the fact that the signal wavefront is curved is explicitly built into the processor. This then permits the radius of curvature of the wave to be estimated, where the radius is then identified as the range of the source.

The third case is the localization of an acoustic source in an oceanic waveguide. Here, the Normal-Mode Model [4] of propagation is used to localize a narrow-band source in the ocean. In this example, the modes and the modal coefficients are jointly estimated along with the source coordinates, thereby eliminating the so-called “mismatch problem” [5] that plagues matched-field processing.

All of these examples are based on a recursive Kalman-type estimation scheme, since this formalism allows the physical models to be incorporated in a natural manner [3]. Results using experimental data are presented for all three cases.

2. BEARING ESTIMATION

Consider an acoustic line array of N equally-spaced hydrophones on the x-axis of an x-y coordinate system to be moving in the +x direction. Further, assume that a plane wave, emanating from a far-field source with a frequency of ω_0 is impinging on the array at a bearing angle of θ measured from broadside, *i.e.,* the y-axis. The signal on the n^{th} hydrophone is then given by

$$p_n = ae^{-ik(nd+vt)\sin\theta + i\omega_0 t}\,. \tag{1}$$

Here, $k = \omega_0 / c$, is the wavenumber, with c being the speed of sound, v is the speed of the array along the x-axis, t is time and d is the spacing of the hydrophone receivers. If we now define a state vector as

$$X = [a \ \theta \ \omega_0]^T , \tag{2}$$

with T signifying the transpose, and consider Equation 1 as a measurement vector element, then a predictor-corrector form of a Kalman filter [3] can be identified, with Equation 2 as the state equation and a measurement equation given by

$$Y(a,\theta,\omega_0) = [p_1 \ p_2 \cdots p_N]^T . \tag{3}$$

The measurement equations are nonlinear functions of the states, so that the extended Kalman filter or EKF must be used [3]. An experiment was carried out in the Baltic by FOI using a narrowband source at a depth of about 40 meters. The array was towed past at a nominal range of 500 meters. The bearing was estimated using the system described by Equations 2 and 3 [6] and the results are depicted in Fig 1. The signal-to-noise ratio (SNR) at the hydrophone level is approximately 0 dB and the frequency is 121 Hz. The bottom curve is the bearing estimate by the full 6 wavelength aperture of the array using a conventional beamformer. The dots are the results using the same beamformer, but using only two wavelengths of aperture. The solid lines at the top are the results using the model-based bearing estimator using the two wavelength segment.

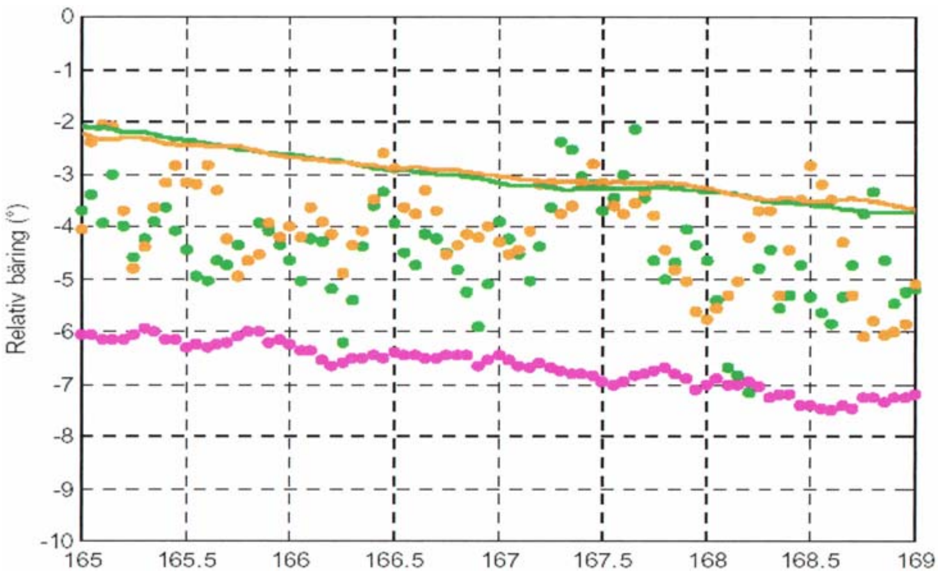

Fig.1: *Towed array bearing estimation results. The horizontal axis is time in seconds and the vertical axis is the bearing in degrees. The nominal bearing is broadside to the array and the tow speed is 4 kts.*

The success of this experiment motivated the development of a broadband version of such an estimation scheme. In principal, this could be done by a parallel set of narrowband procedures as described above, but this would require a prohibitive amount of computational time. Instead, a broadband processor was developed based on a frequency domain representation and is described in Reference 7. This has not been evaluated yet with actual at-sea data, but we show results for simulated broadband data in Fig. 2. The SNR is -15 dB and the mean acoustic aperture is one wavelength for a frequency band of 100-400 Hz.

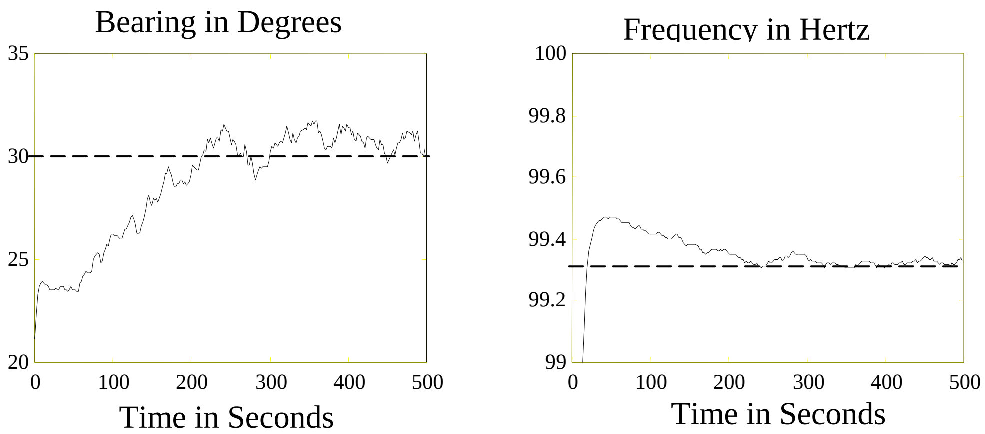

Fig. 2: *Broadband bearing estimation results. The 5 element array was moving with a speed of 10 meters/second. The bearing was initialized at 24 degrees and the frequency was initialized at 99 Hz, where the source fundamental frequency was 100 Hz. The dotted lines indicate the true values.*

3. WAVEFRONT CURVATURE RANGING

If, instead of a plane wave, suppose that a circular wavefront is arriving at the array at bearing angle θ. For the two-dimensional case, the time delay of the wave with respect to the n^{th} element is found, with the help of the law of cosines, to be

$$\tau(R,\theta) = (R - R_n)/c = R\left(1 - \left[1 + \left(\{x_n + vt\}/R\right)^2 - 2\left(\{x_n + vt\}/R\right)\sin\theta\right]^{1/2}\right). \qquad (4)$$

Here, R is the range to the origin of the x-y coordinate system and R_n is the range to the n^{th} element. The measurement at the n^{th} element is then

$$p_n(a,\theta,\omega_0,R) = ae^{i\omega_0(t-\tau(R,\theta))}. \qquad (5)$$

Defining the state equation as

$$X = [a \ \theta \ \omega_0 \ \ R]^T \ , \tag{6}$$

and the measurement equation as

$$Y(a, \theta, \omega_0, R) = [\, p_1 \ p_2 \cdots p_N \,]^T \ , \tag{7}$$

the state vector and therefore the range and bearing can be jointly estimated. An example of range only estimation is shown in Fig 3. This result is based on the same data as used for the plane wave results shown in Fig. 1.

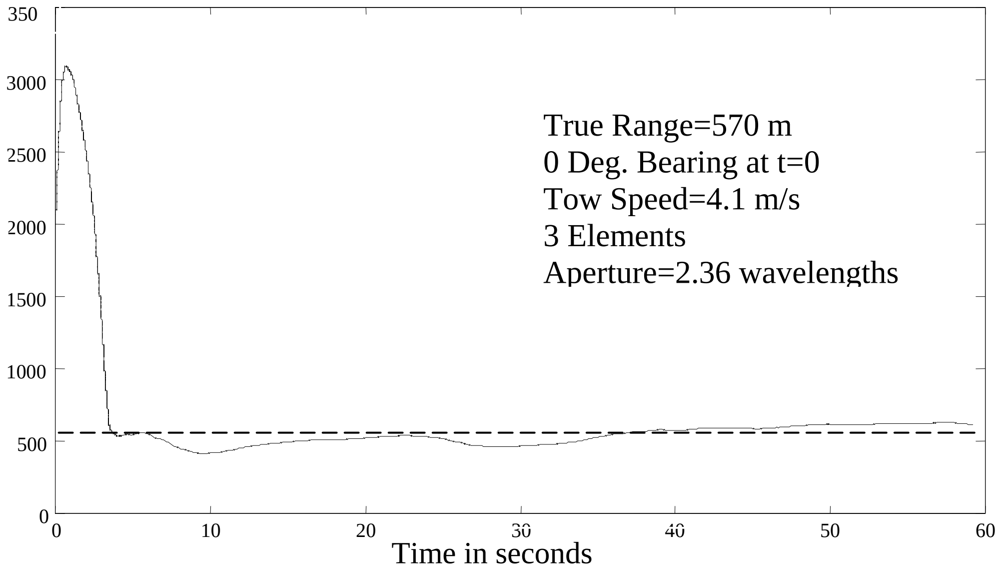

Fig. 3: *Narrowband rang estimation results. The 3 element array was moving at a speed of 4 kts. The range, which is indicated on the vertical axis, was initialized 1000 meters.*

4. SOURCE LOCALIZATION

In the Matched–Field approach to source localization, unless the model parameters are known to a sufficient degree of accuracy, the estimation process fails [5]. MFP is carried out by using the model to predict the measurements on the array for an exhaustive set of source coordinates. These predictions are then compared or “matched” to the measurements, and the source coordinates associated with the best match are deemed to be the estimated position. By formulating the localization problem as a Kalman-based recursive estimator, the mismatch problem can be dealt with by including the troublesome model parameters as additional unknown parameters, *i.e.*, by “augmenting” them into the state vector [8,9].

In this example, the measurement equation is based on the normal-mode propagation model. Thus,

$$p_n(r_s, z_s) = \sum_{m=1}^{M} H_0(k_m r_s)\phi_{m,1}(z_s)\phi_{m,1}(z_n) = \sum_{m=1}^{M} \beta_m(r_s, z_s)\phi_{m,1}(z_n)\,. \tag{8}$$

Here, p_n is the pressure measurement on the n^{th} hydrophone of the vertical array, r_s and z_s are the respective source range and depth, z_n is the depth of the n^{th} hydrophone, k_m is the horizontal wavenumber for the m^{th} mode and $\phi_{m,1}(z)$ is the m^{th} modal function evaluated at the depth z. The state equation evolves from the so-called vertical equation, *i.e.,* the total differential equation for the modal functions resulting from the separation of variables procedure on the wave equation. We write this equation explicitly as

$$\frac{d^2}{dz^2}\phi_m(z) + \gamma_m^2(z)\phi_m(z) = 0\,. \tag{9}$$

By defining the two-dimensional state-space $\varphi_m(z) = [\phi_{m,1}(z)\ \phi_{m,2}(z)]^T$, a two-dimensional subsystem (state-space) for the m^{th} mode evolves as

$$\frac{d}{dz}\varphi_m(z) = A_m(z)\varphi_m(z) \quad or \quad \frac{d}{dz}\begin{bmatrix}\phi_{m,1}(z)\\ \phi_{m,2}(z)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\gamma_m^2(z) & 0\end{bmatrix}\begin{bmatrix}\phi_{m,1}(z)\\ \phi_{m,2}(z)\end{bmatrix}, \tag{10}$$

where $\phi_{m,1}(z) = \phi_m(z)$ and $\phi_{m,2}(z) = \frac{d}{dz}\phi_{m,1}(z)$. For M modes, the state vector is $2M \times 1$ dimensional and the full state equation is then

$$\frac{d}{dz}\varphi(z) = A_\varphi(z)\varphi(z) = \begin{bmatrix}A_1(z) & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & A_M(z)\end{bmatrix}\begin{bmatrix}\varphi_1(z)\\ \vdots\\ \varphi_M(z)\end{bmatrix}. \tag{11}$$

The source parameters are now found as follows. Identifying Equation 8 as the measurement equation, a state equation is written based on an augmented state vector defined as

$$\bar{x} = [\varphi_1(z)\ \varphi_2(z)\cdots\varphi_M(z)\ \beta_1(z)\ \beta_2(z)\ \cdots\beta_M(z)]^T = [\varphi(z)\ \mathrm{B}(z)]^T\,. \tag{12}$$

By approximating the differential equation in Equation 12 using discrete first differences, the state vector is sequenced along the measurement array using

$$\begin{bmatrix} \hat{\varphi}(z\,|\,z) \\ \hat{\mathrm{B}}(z\,|\,z) \end{bmatrix} = \begin{bmatrix} (I + \Delta z A_{\varphi}(z)) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \hat{\varphi}(z\,|\,z-\Delta z) \\ \hat{\mathrm{B}}(z\,|\,z-\Delta z) \end{bmatrix} \tag{13}$$

Unlike usual sequential processing schemes, where the sequencing takes place over time samples, here the sequence is over the spatial samples provided by the vertical line array. Thus, the measurement equation is a scalar- -- a great computational savings!

Having the state vector estimate, and the propagator of Equation 13, the source coordinates are then found by a nonlinear least squares estimate. That is, the squared error J is minimized using the *polytope* method [10], where

$$J(r_s, z_s) = \sum_{m=1}^{M} [\hat{\beta}_m - H_0(k_m r_s)\phi_{m,1}(z_s)]^2 \tag{14}$$

The results for a case of at sea data are shown in Fig. 4 . The data for this example is from the Hudson Canyon experiment [11].

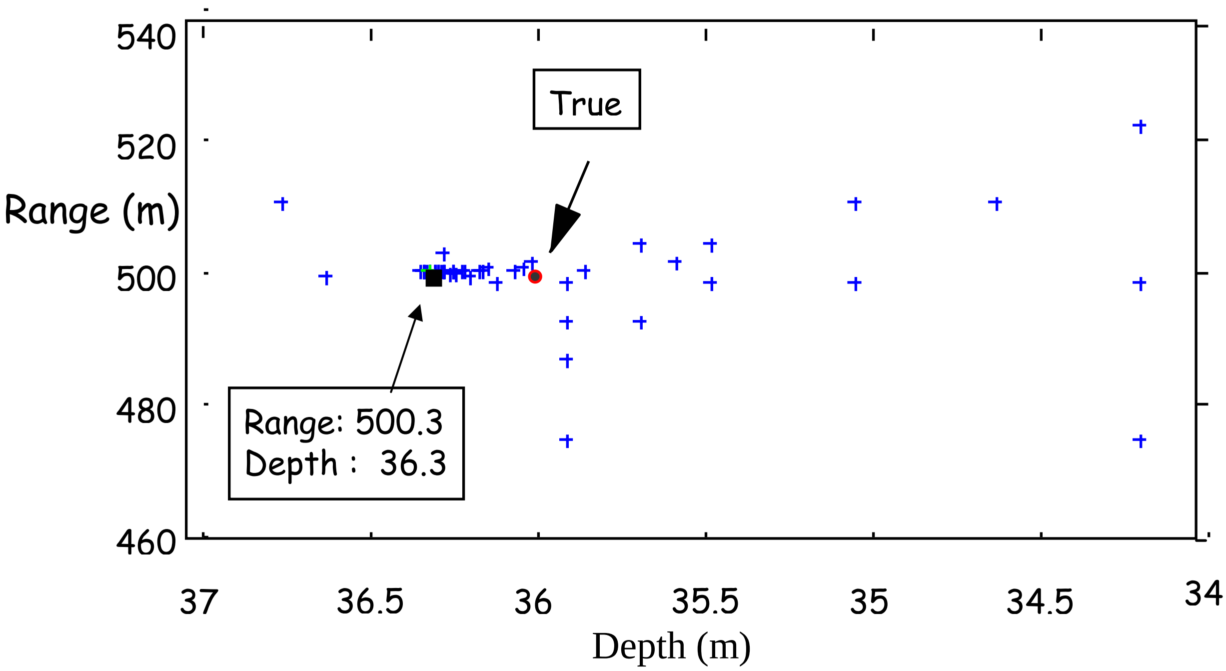

Fig. 4: *Narrowband localization results. The 50 Hz source was located at a depth of 36 meters and a range of 0.5 km from the fixed vertical array. The element spacing was 2.5 meters.*

5. SUMMARY

The results in all three cases make clear that the inclusion of the models provides significant improvement in performance. In the first case, that of bearing estimation, the performance of the model-based processor clearly provides a much smaller bearing estimation error when compared with the conventional beamformer. In the case of wavefront curvature, although here there is no direct comparison to any other method, we note that the results are based on a SNR of 0 dB and a range-to-aperture ratio of 19.5. Furthermore, this solution required only 3 seconds to converge. The localization case also is not directly compared to any other case in a quantitative manner, but we note two points. First, the solution is highly accurate. Second, unlike the matched-field approach, an exhaustive search over the parameter space was not necessary.

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